

# Dynamic Semantics and Ambiguity

## Abstract

Most work on ambiguity focuses on the semantic representation of single ambiguous sentences. We present a dynamic semantics which gives a formal account of the behavior of ambiguous expressions occurring in a sequence of sentences. It is considered how ambiguity survives throughout a discourse and how contextual information can restrict ambiguity.

## 1 Introduction

Natural language expressions can be highly ambiguous, and this ambiguity may have various faces. Well-known phenomena include lexical and syntactic ambiguities. In this paper we focus on constructing a discourse semantic for quantificational ambiguities and ambiguities evolving in pronoun resolution, as exemplified in (1.a) and (1.b), respectively.

- (1) a. Every man loves a woman.
- b. A man saw a boy in the park. He whistled.

The different readings of (1.a) correspond to the two logical representations in (2) and the pronoun in (1.b) can be resolved in two ways as illustrated in (3)

- (2) a.  $\forall x (man(x) \rightarrow \exists y (woman(y) \wedge love(x, y)))$ .
- b.  $\exists y (woman(y) \wedge \forall x (man(x) \rightarrow love(x, y)))$ .
- (3) a. A man<sub>i</sub> saw a boy in the park. He<sub>i</sub> whistled.
- b. A man saw a boy<sub>i</sub> in the park. He<sub>i</sub> whistled.

We refer the reader to [vDP96] for extensive discussions of these and other examples of quantificational ambiguity. Often ambiguous sentences have a preferred reading, cf. [KM93, JK96] for more details, but adding preference selection would result in a nonmonotonic framework. For instance, assume that a wide scope reading for the universal quantifier is preferred in (1.a), then it entails (2.a). On the other hand, if (1.a) is followed by (4) it does not entail (2.a) anymore.

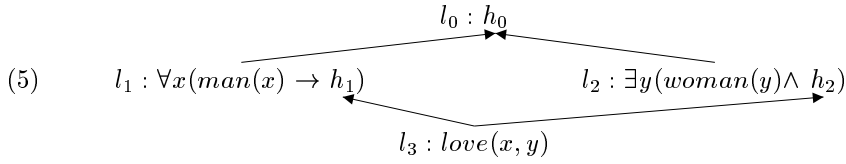
- (4) But she is already married.

Clearly, if we want to process a discourse from left to right in a monotonic fashion, our semantic representation for (1.a) must initially allow for both possibilities.

The problem of ambiguity and underspecification has recently enjoyed a considerable increase in attention from computational linguists, computer scientists and logicians (see, for instance, [vDP96]), but the focus has mostly been on giving a semantic representation for single sentences. Although some of the theories are couched in a dynamic setting (cf. for example [Bos95, Rey93]), they do not give a formal account of the behavior of ambiguous expressions in a discourse. Roughly, two things are lacking: First, there is no detailed account of pronoun resolution. In [Bos95] coindexation is assumed and in UDRT ([Rey93, Rey95]) only one way of binding is considered, but not different ways of binding in parallel. Second, there is no notion of an ambiguous context and ambiguous updating. In other words, the update potential of ambiguous sentences is not given.

The aim of this paper is to give a dynamic semantics for a language that includes ambiguous expressions, where ambiguity is represented in a non-redundant, underspecified way, as it is done in [Rey93, Bos95].

It is possible to represent quantificationally ambiguous formulas in a compact way as upper semi-lattices, cf. [Rey93]:



**Definition 1 (Underspecified Representation)** An underspecified representation ( $UR$ ) is a quadruple  $\langle LHF, L, H, C \rangle$ , where  $LHF$  is the set of labeled h-formulas, and  $C$  is a partial order on the labels ( $L$ ) and holes ( $H$ ) occurring in  $LHF$ . A *hole* indicates that the concrete form of this subformula is not known yet. For instance we do not know what the scope of the universal quantifier is, so the succedent of the implication is a hole. *h-formulas* are the formulas possibly contain holes. In (5)  $LHF = \{l_0 : h_0, l_1 : \forall x(man(x) \rightarrow h_1), l_2 : \exists y(woman(y) \wedge h_2), l_3 : love(x, y)\}$  and  $C = \{l_1 \leq h_0, l_2 \leq h_0, l_3 \leq h_1, l_3 \leq h_2\}$ .

## 2 A Dynamic Semantics for Ambiguity

To combine underspecified representation and update semantics we take Dekker's *EDPL* [Dek93] as a basis and adapt it in a way such that it fits our purposes. We chose *EDPL* because it is both, eliminative *and* distributive and the notion of an information state is explicit.

The adaption proceeds mainly along two lines. First, contrary to approaches in the tradition of dynamic predicate logic (DPL), see [GS91], we do not take resolution of pronouns as granted; i.e. coindexation of pronouns and their antecedents is not presupposed. We dispense with this — admittedly quite handy — restriction on our input, because it is unrealistic and anaphora resolution is an important source of ambiguity in natural language texts, which we would like to account for. If we do not assume coindexation, it is necessary to deal with pronouns in a different way. We do so by binding pronominal arguments by three new operators *he*, *she* and *it*, similar to [Bea93], that roughly say that its arguments have to be identified with an accessible antecedent in a given context. Second, whereas in *EDPL* contexts are identified with states, we have to think about the notion of an ambiguous context. In this paper we represent ambiguous contexts,  $\sigma$ , simply by sets of states, where each state in a set of states represents an unambiguous reading of the context. Of course the notion of an update has to be redefined appropriately.

Before adapting *EDPL*, we briefly repeats its semantic definitions.

### Definition 2 (Semantics of EDPL)

$$\begin{aligned} s \llbracket R(x_1..x_n) \rrbracket &= \{i \in s \mid \langle i(x_1)..i(x_n) \rangle \in F(R)\} \text{ if } x_1..x_n \in D(s) \\ s \llbracket x = y \rrbracket &= \{i \in s \mid i(x) = i(y)\} \text{ if } x, y \in D(s) \\ s \llbracket \neg\varphi \rrbracket &= s - s \llbracket \varphi \rrbracket \\ s \llbracket \exists x\varphi \rrbracket &= s[x] \llbracket \varphi \rrbracket \text{ if } x \notin D(s) \\ s \llbracket \varphi \wedge \psi \rrbracket &= s \llbracket \varphi \rrbracket \llbracket \psi \rrbracket \end{aligned}$$

where  $s - s' = s \setminus \{i \mid_{D(i)} \mid i \in s'\}$  and  
 $s[x] = \{j \mid \exists i \in s : i \leq_{\{x\}} j\}$ , such that  $i \leq_X j$  iff  
 $D(j) = D(i) \cup X$  and  $i = j \mid_{D(i)}$

The other connectives,  $\forall$ ,  $\vee$ ,  $\rightarrow$ , can be defined in terms of the definitions given above.

### 2.1 Pronoun Resolution

The first difference between *EDPL* and our dynamic predicate logic with ambiguity (*DPLA*), concerns the treatment of pronouns. In *EDPL* it is assumed that pronouns are resolved and therefore pronouns and there antecedents use the same variable. In our approach pronouns are bound by a pronominal operator  $P$ ,  $P \in \{\text{he, she, it}\}$ , so *she*  $x\varphi$  means that the argument  $x$

in  $\varphi$  is pronominal and its antecedent must be female.<sup>1</sup> Because pronominal resolution might be ambiguous, updating a context  $\sigma$  with  $\llbracket Px\varphi \rrbracket$  does not have to return a single update state for each  $s \in \sigma$ , but a set of them.

**Definition 3 (Pronoun Resolution)** Let  $\sigma$  be an ambiguous state.

$$\sigma \llbracket Px\varphi \rrbracket = \bigcup_{s \in \sigma} s \llbracket Px\varphi \rrbracket, \quad \text{where } P \in \{\text{he, she, it}\}$$

It is obvious that this way of resolving pronouns is very inefficient, but we want to give a monotone semantics for ambiguity, and of course it is possible to augment our theory with some heuristics that allow to restrict the set of readings or ways of resolving pronouns, see for instance [JK96].

## 2.2 Quantificational Ambiguities

Next we consider quantificational ambiguities, for which a compact representation was given earlier. The semantics of underspecified representations is defined in terms of total disambiguations see [Rey95], so avoiding redundancy on the syntactic side is possible, but it returns on the semantic side. Often this is not a problem, because one can consider the syntactic side, only. For instance given a complete and sound calculus, reasoning in this system amounts to syntactic manipulation. But in an ambiguous setting things are a bit different: underspecified representations are compact representations of several formulas, i.e. they are syntactically ambiguous, and at the moment it is not obvious how to reason with ambiguous formulas without disambiguating them at least partially, cf. [Jas97, Mdr98].

The question is how can we restrict the massive branching of contexts when updating with ambiguous information. If we do not update with total disambiguations but instead interleave updating and disambiguation we might gain a decrease in complexity because further disambiguation steps depend on earlier successful updates. Updates of complex formulas are computed in a top-down manner. The first step is to identify the possible outermost operators in a quantificationally ambiguous expression and start computing the updates until we face a *hole*. Then we make a choice again and check what are the operators which may have the next wider scope respectively, continue updating, and so on. Possibly some of these first steps exclude some disambiguations because they are not compatible with the given input context.

Partially updating with quantificationally ambiguous material changes also the set of ordering constraints. This makes it necessary to make contexts more complex. We will say that an ambiguous context  $\sigma$  is a set of triples of the form  $\langle s, C, LHF \rangle$ . Updating  $\sigma$  proceeds in a top-down manner:

**Definition 4** Let  $UR = \langle LHF, L, C \rangle$  be an underspecified representation.  $\text{top}(UR)$  returns the formula of the top node and  $\text{dtrs}(h)$  are the possible immediate subformulas of a hole  $h$ .

$$\begin{aligned} \sigma \llbracket UR \rrbracket &= \{ \langle s, C, LHF \rangle \mid \exists C', LHF' : \langle s, C', LHF' \rangle \in \sigma \} \llbracket \text{top}(UR) \rrbracket \\ \sigma \llbracket h \rrbracket &= \bigcup_{l_j : \varphi_j \in \text{dtrs}(h)} \{ \langle s, C \cup \{l_k \leq h'\}, LHF \rangle \mid \langle s, C, LHF \rangle \in \sigma, l_k : \psi \in \text{dtrs}(h) \} \llbracket \varphi_j[h'] \rrbracket \\ \sigma \llbracket h \rrbracket &= \sigma \llbracket \varphi_j \rrbracket, \text{ if } \text{dtrs}(h) = \{l_j : \varphi_j\} \text{ and } \varphi_j \text{ does not contain a hole} \end{aligned}$$

Having defined the update function of underspecified representations and expressions containing pronouns, we adapt the update conditions of the kind of expressions already considered in *EDPL*. First we have to adapt the update definition of unambiguous contexts from simple states to triples of the form  $\langle s, C, LHF \rangle$ .

**Definition 5** if  $\varphi$  does not contain holes or underspecified representations:

$$\langle s, C, LHF \rangle \llbracket \varphi \rrbracket = \begin{cases} \langle s \llbracket \varphi \rrbracket, C, LHF \rangle, & \text{if } s \llbracket \varphi \rrbracket \neq \emptyset \text{ and defined and } C \not\vdash \perp \\ \text{undefined,} & \text{otherwise} \end{cases}$$

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<sup>1</sup>This is of course a simplified picture of the whole issue of grammatical and natural gender and the way it restricts pronoun resolution.

**Definition 6 (Semantics of DPLA)** The update potentials of the logical connectives are those given in Definition 3 plus the following ones, where  $s^c$  is of the form  $\langle s, C, LHF \rangle$ .

$$\begin{aligned}
\sigma[R(x_1 \dots x_n)] &= \{s^c[R(x_1 \dots x_n)] \mid s^c \in \sigma, s^c[R(x_1 \dots x_n)] \text{ is defined}\} \\
\sigma[x = y] &= \{s^c[x = y] \mid s^c \in \sigma, s^c[x = y] \text{ is defined}\} \\
\sigma[\neg\varphi] &= \{s^c - t^c \mid s^c \in \sigma, t^c \in \{s^c\}[\varphi]\} \\
\sigma[\exists x\varphi] &= \{s^c[\exists x\varphi] \mid s^c \in \sigma, s^c[\exists x\varphi] \text{ is defined}\} \\
\sigma[\varphi \wedge \psi] &= \sigma[\varphi][\psi] \\
\sigma[\mathbf{P}x\varphi] &= \bigcup_{s^c \in \sigma} s^c[\mathbf{P}x\varphi], \quad \text{if defined}
\end{aligned}$$

where  $\langle s, C, LHF \rangle - \langle t, C, LHF \rangle = \langle s - t, C, LHF \rangle$

## 2.3 Some Properties of DPLA

*EDPL* is distributive and eliminative; but what about *DPLA*? It is easy to see that distributivity holds, because in none of our definitions the update depends on the whole context. Combining this with the fact that *EDPL* is distributive we get:

**Observation 1**  $\sigma[\varphi] = \bigcup_{s^c \in \sigma} \{s^c\}[\varphi]$ , if defined

Although updates of ambiguous formulas possibly contain more readings than the original context, each of these reading eliminates some (possibly null) assignment functions.

**Observation 2** *DPLA* is eliminative.

For the fragment which does not involve ambiguities the properties of *EDPL* generally hold for *DPLA*, too.

**Definition 7 (Entailment)** Following [Dek93], where  $s \leq s'$  iff  $\forall i \in s \exists j \in s' : i \subseteq j$  we say that  $\sigma \leq \sigma'$  iff  $\forall \langle s, C, LHF \rangle \forall \langle s', C', LHF' \rangle : s \leq s'$ . The entailment relation can now be defined as:

$$\varphi_1 \dots \varphi_n \models \psi \text{ iff } \sigma[\varphi_1] \dots [\varphi_n] \leq \sigma[\psi]$$

This is just one of many possible definitions of the entailment relation. Two things should be pointed out: first, the dynamic force of the existential quantifier does not exceed the entailment relation. So anaphora occurring in the conclusion cannot be bound by an existential quantifier in the premisses. Second, our notion of entailment is very weak, because it presupposes that all readings in the premises entail all readings in the conclusion, but for dealing with natural language semantics it seems to be an appropriate definition, see [vD96, MdR98, Rey95] for further discussion and refinements.

## 3 Conclusion

Our formalism is monotone and it inherits the properties of eliminativity and distributivity from *EDPL*. It is more general than *DPL* based approaches, because it gives an account for pronoun resolution without relying on coindexation and it does so by considering different possibilities in parallel. Contrary to [Bos95, Rey93] the update potential of ambiguous sentences is defined properly and it is possible to compute the update of a sequence of sentences.

*DPLA* allows us to give a general picture of ambiguous updating, which can be extended in several directions by adding some heuristics to yield a more efficient formalism. We would like to point out, that we did not try to model how human speakers deal with ambiguous information, but we did try to give an account that is general enough to be specified in a way to do so.

It might be interesting to see how our framework interacts with approaches on deduction in an ambiguous setting, cf. [Jas97, MdR98]. Jaspars noticed that the notion of relative ambiguity is necessary in order to get a well behaved calculus for ambiguity. This notion is part of our framework and of dynamic semantics in general, because updates depend on the context to which they are applied, and so do ambiguous updates, too. The next step will be to see how a calculus for reasoning in ambiguous dynamic semantics can be developed.

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