

# A vector space semantics for dimensional adjectives

- Abstract for ESSLLI'98 Student Session -

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This paper develops a semantic analysis of dimensional adjectives (DAs) in predicative position of the kind exemplified in (1)-(3) within the framework of vector space semantics (VSS) ([1], [2]).

- (1) Margaret is (3cm) taller than Carmen.
- (2) The board is (10cm) longer than the table is wide.
- (3) The board is (100cm) long.

(1)-(2) are comparatives, (3) contains a DA in the absolute. I disregard equatives (*as tall as*) and superlatives (*(the) tallest*) in this talk.

Comparatives and absolutes have been discussed extensively in the formal semantic literature (see [3]). Nevertheless, some questions have remained unanswered or have received only unsatisfactory explanations. Among those are the compositional analysis of measure phrases (MPs), as in (1)-(3), and of so-called comparatives of deviation, to be illustrated below.

While semantic theories analyzing comparatives usually do include MPs on the representational level, it is often not made explicit how they can be incorporated compositionally (exceptions are [6], [7], which however do not compositionally analyze all phenomena discussed in this paper). The problem for most accounts is that they require that an MP has access to the denotations of both the complement of *than* (*Carmen* in (1)) and the matrix subject NP (*Margaret* in (1)). Given the standard syntactic analysis of adjective phrases that is generally assumed ([MP [ [more [ ADJ ] ] XP ]], where XP is the (extraposed) *than*-phrase), this is not straightforward. It will be shown that the VS semantics developed for simple comparatives and absolutes allows for a compositional incorporation of MPs, using the semantics [1] and [2] present for MPs in locative PPs (*The tree is ten meters outside the house*).

Comparison of deviation is best illustrated with examples that don't allow a regular comparative interpretation. Given the information that the resting heart rate (RHR) of an average person is between 68 and 72 beats per minute (bpm), and that a RHR below 68bpm is considered low, and a RHR above 72bpm high, (4) can felicitously be uttered.

- (4) Terry's RHR is lower than Jo's RHR is high.

However, (4) cannot be interpreted to mean that Terry's absolute RHR is lower than Jo's absolute RHR (though in the given context this would follow). It can only be interpreted to mean that the extent to which Terry's RHR is below the lower limit of the standard (68bpm) is greater than the extent to which Jo's RHR exceeds the upper limit (72bpm).<sup>1</sup> That is,

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<sup>1</sup>I should mention that speakers' judgments on such examples vary greatly. The differences seem to have to do with how salient the standards are and how used a speaker is to dealing with such complex comparisons.

what is being compared is the *deviation* of the two objects from the standards for the given dimension. This interpretation is also available for sentences that do allow the regular reading, though it might be less accessible. Thus, examples like (2) without the MP are in fact ambiguous between a regular comparative reading and a deviation reading.

Most accounts of comparatives only mention comparison of deviation in passing, at best a sketch of an analysis is presented ([4]). To my knowledge, the analysis presented in this paper is the first compositional account of this phenomenon.

VSS is a framework in the tradition of Montague grammar with vectors as ontological primitives. It was originally developed by [1] and [2] to account for locative PPs. Besides presenting a compositional analysis of comparatives, one main goal of this talk is to show that VSS can fruitfully be applied to other areas as well. Comparatives seem particularly promising in this respect, as it has been established that many languages employ the same syntactic devices to encode both spatial and comparative relations (see [5]). Not only that, many languages use the same mechanisms to encode temporal relations. For this reason, a framework that captures the similarities between locatives, comparatives and temporals is preferable to one that is only suitable to account for one area, particularly if the semantic relationships between these phenomena are to be studied.

### Sketch of Analysis

In VSS, vectors are ontological primitives. The operations vector addition  $+$ , scalar multiplication  $\cdot$ , and vector norm  $|\cdot|$ , are defined in the usual way ([1],[2],[8]).

DAs make reference to dimensions such as HEIGHT. Each dimension is associated with a scale, more precisely with respect to the DAs studied in this paper, a *ratio* scale. A scale can be defined as a set of located vectors.<sup>2</sup>

A positive DA such as *tall* denotes the set of vectors on the scale that point into the positive direction, a negative adjective such as *short* the set of vectors that point into the negative direction.

For the purposes of this paper, I assume that the compared objects are mapped onto their *eigenvector*, a positive vector on the appropriate scale located at the origin. Consider example (1). *Carmen* is mapped onto the vector  $\mathbf{c}$  in Figure 1. The comparison clause *than Carmen* denotes the set of vectors  $V$  starting at the endpoint of  $\mathbf{c}$ .  $\mathbf{l}$  and  $\mathbf{t}$  are two vectors in that set. The comparative morphemes *more/-er* and *less* are two-place functions that take a DA and the set  $V$  as arguments, and map  $V$  onto a subset  $S$ .  $S$  will differ depending on which comparative morpheme is used and whether the DA is negative or positive. *more/-er* maps  $V$  onto a subset  $S$  such that the directionality of the vectors in  $S$  is as specified by the DA, *less* on the other hand reverses the directionality. That is, *shorter than X* denotes a set of vectors pointing into the negative direction, the vectors in *less short than X* point into the positive direction. For *tall*, it is just the opposite. Figure 1 illustrates *taller/less tall than Carmen*.  $\mathbf{t}$  is in the set *taller than Carmen*,  $\mathbf{l}$  in *less tall than Carmen*.

The subject of predication, *Margaret*, is also mapped onto its *eigenvector*  $\mathbf{m}$  on the height scale. There will be one vector  $\mathbf{v}$  in the set denoted by *taller than Carmen* such that its endpoint coincides with the endpoint of  $\mathbf{m}$ . This is illustrated in Figure 2.

The (somewhat simplified) semantic representation for (1) without the MP is given in (6).

$$(6) \exists \mathbf{v}[\mathbf{m} = \mathbf{c} + \mathbf{v} \wedge \text{HEIGHT}(\mathbf{v})]$$

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<sup>2</sup>Located vectors have fixed end- and start-points in contrast to vectors. Henceforth, I will use the term *vector* to mean located vector.

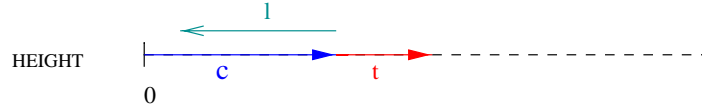


Figure 1: taller/less tall than Carmen

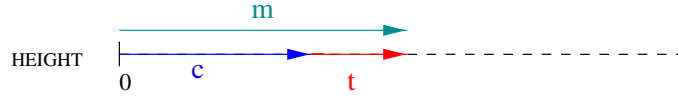


Figure 2: Margaret is taller than Carmen

In the analysis presented here, an MP is a function that applies to  $S$  (the set denoted by *taller than Carmen*) and maps it onto a subset of vectors which have the length specified by MP,  $3cm$  in the example. The MP does not have to access the denotation of *Margaret* at all. Accounts which analyze comparatives as a relation holding between the degrees denoted by the objects don't have this option. The semantic representation for (1) with the MP is (8):

$$(6) \exists \mathbf{v}[\mathbf{m} = \mathbf{c} + \mathbf{v} \wedge \text{HEIGHT}(\mathbf{v}) \wedge |\mathbf{v}| = 3cm]$$

The semantics for absolutes such as (3) is similar. Intuitively, a board is long if it exceeds a contextually determined standard  $\mathbf{s1}$ , and short if it is below a standard  $\mathbf{ss}$ , where  $\mathbf{s1}$  and  $\mathbf{ss}$  can be different. Thus, the representation for (3) is as in (9) ( $\mathbf{b}$  is the *eigenvector* of the board).

$$(9) \exists \mathbf{v}[\mathbf{b} = \mathbf{s1} + \mathbf{v} \wedge \text{LENGTH}(\mathbf{v})]$$

If there is an MP, the reference to a standard disappears. A board that is 100cm long is not necessarily long. This is captured by having an MP set all standards to  $\mathbf{0}$ . This results in the vector  $\mathbf{v}$  being identical to the *eigenvector*  $\mathbf{b}$ .

$$(9) \exists \mathbf{v}[\mathbf{b} = \mathbf{0} + \mathbf{v} \wedge \text{LENGTH}(\mathbf{v}) \wedge |\mathbf{v}| = 100cm]$$

Only positive adjectives can be modified by MPs in the absolute such that the MP specifies the extension of the measured object:

(10) The board is 100cm long.

(11) # The board is 100cm short.

Given the semantics above, applying an MP to a negative adjective will require there to be a vector  $\mathbf{v}$  which starts at the origin and points into the negative direction. For scales without a negative extension such as the scale of length, there is no such vector. This in itself might be enough to explain the anomaly of (11). But we can also cause the semantic composition to fail by giving an interpretation to negative adjectives that requires the standard  $\mathbf{s}$  to be greater than  $\mathbf{0}$ . This captures better the observation that negative adjectives are always used with reference to a standard. For example, some speakers of English report that *The board is 100cm short* is acceptable, but that it can only mean that the board is 100cm *too* short with respect to some standard.

The semantics of comparison of deviation is rather intricate. As should be clear from the paraphrase given above for (4), one first has to calculate the deviation of the objects from the standards. Next, the length of the deviation vectors  $\mathbf{v}$  and  $\mathbf{w}$  has to be compared. For this, the two vectors themselves are mapped onto their *eigenvectors*  $\mathbf{v}'$  and  $\mathbf{w}'$  on a scale of length. This is illustrated in Figure 3 for (4) (where  $\mathbf{sl}$  is the standard for a low RHR,  $\mathbf{sh}$  the standard for a high RHR, and  $\mathbf{t}$  and  $\mathbf{j}$  are the *eigenvectors* for Terry’s and Jo’s RHRs).

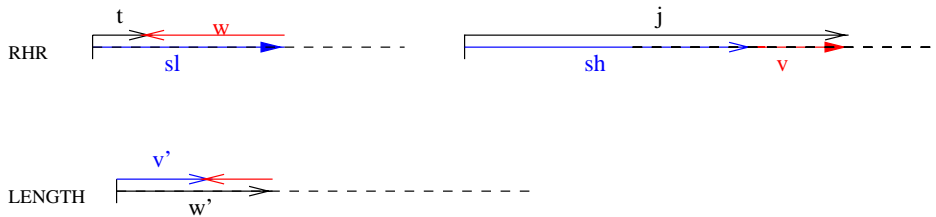


Figure 3: Comparison of deviation

Thus, comparison of deviation involves two scales. As for regular comparatives, a fully compositional semantics is presented in the paper.

### Summary

In summary, the paper demonstrates that VSS lends itself easily for analyzing comparatives. Future work will have to show that it can also be adapted to account for temporal relations. A fully compositional semantics in the style of Montague grammar is presented for regular comparatives, including MPs and comparatives of deviation. The full version of the paper furthermore compares the present analysis with the account developed in [4] for the phenomena discussed. In particular with respect to the unavailability of the regular comparative interpretation for sentences like (4), the two accounts make different empirical predictions. Time and space permitting, I will address this issue in the paper. [4] explains the infelicity of (4) with respect to the regular interpretation in terms of the opposite polarity of the two adjectives involved. I, on the other hand would like to argue, following [6], that the regular reading should be ruled out on the basis of Gricean-style principles such as *don't be redundant*. Since the two adjectives in (4) have identical scale reference, the second one is redundant unless it provides some additional information such as the second standard needed for comparison of deviation. Such an analysis makes interesting predictions which will have to be studied in future research. For instance, it would predict that (12), which contains two adjectives of opposing polarity and different scale reference, and which would be judged infelicitous on the regular interpretation by [4], is in fact felicitous.

(12) The ski poles are shorter than the box is wide.

I thus hope to show that a compositional VS semantics can serve as an ideal basis for future empirical work both in terms of its predictions regarding comparatives, and in terms of its ability to draw together semantic analyses of a range of different empirical phenomena.

## References

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